

# ECONOMIC BASE AND INPUT-OUTPUT MULTIPLIERS: AN EMPIRICAL LINKAGE

*Rene C. Hinojosa and Bruce William Pigozzi\**

## Introduction

Recently the notion of mathematical equivalence of economic base and aggregate input-output multipliers has been resurrected [8]. The mathematical presentations have been convincing, but they have lacked an empirical verification or demonstration. This paper, using national and Washington state data and an alternative input-output formulation of the sectoral multipliers, reconstructs the base multiplier as a weighted aggregate, thus providing a benchmark against which estimates of base multipliers derived from various techniques such as location quotients may be assessed.

## The Model

Hinojosa and Pigozzi [5, 6, 10], while pursuing an alternative nonsurvey technique for updating input-output tables, developed a partitioning of employment that leads to the classic basic/nonbasic in a productive manner. Their formulation is summarized as follows. First the classic economic input-output formulation of  $n$  industries is provided in:

$$X = (I-A)^{-1}Y = BY \quad (1)$$

We assume a constant employment-output relationship given as:

$$\pi_j = E_j/X_j \quad (2)$$

This enables us to formulate the usual employment multiplier for a sector  $j$  as:

$$K_j = (1/\pi_j) \sum_i (b_{ij}/\pi_i) \text{ for } i = 1 \text{ to } n \quad (3)$$

Now, we can produce a scalar, the product:

$$\Pi X = \pi_1 X_1 + \pi_2 X_2 + \dots + \pi_n X_n = E_1 + E_2 + \dots + E_n \quad (4)$$

$\Pi X = E_t$  or total interindustry employment.

With (1), we can see:

$$\Pi X = \Pi BY. \quad (5)$$

Expanding this matrix product:

$$E_i = \sum_j (\pi_i b_{ij} Y_j) \quad (6)$$

or in matrix form

$$E = \Pi_d B Y$$

where  $\Pi_d$  is the diagonal matrix of  $\pi_i$  terms.

We may also define the column sum of  $\Pi B Y$  as:

$$G_j = \sum_i (\pi_i b_{ij} Y_j) = Y_j \sum_i (\pi_i b_{ij}) \quad (7)$$

or in matrix form

$$G = \Pi B Y_d$$

where  $Y_d$  is the diagonal matrix of final demand. Hence,

$$G_1 + G_2 + \dots + G_n = E_t = E_1 + E_2 + \dots + E_n. \quad (8)$$

The interpretation of  $G_j$  is "the total employment generated in the economy (across all sectors) by delivering  $Y_j$  to final demand. In other words, it measures the overall employment effort required from the region to meet the final demand for the products of sector  $j$ " [6, pp. 270-271].

We can see from (3) that

$$\sum_i (b_{ij} \pi_i) = \pi_j K_j$$

Hence, with (7):

$$G_j = Y_j \pi_j K_j \quad (9)$$

Thus,

$$E_t = \sum_j (G_j) = \sum_j (Y_j \pi_j K_j) \quad (10)$$

It has been shown [6, p. 273] that  $G_j$  can be divided into  $\pi_j Y_j$ , a portion attributed to the direct delivery by sector  $j$  to final demand, and an indirect portion of employment that depends on the degree of

interdependency with the rest of the economy. Thus, we have a measure of total interindustry employment,  $\sum_j(Y_j\pi_jK_j)$ , and a measure of total direct employment,  $\sum_j(Y_j\pi_j)$  that, when placed in ratio form, produces the first hint of an employment base-type multiplier:

$$M_1 = \sum_j(Y_j\pi_jK_j) / \sum_j(Y_j\pi_j) \tag{11}$$

This is essentially a weighted sum of the input-output sector multipliers,  $K_j$ .

It is not possible or appropriate to identify firms or sectors that are exclusively export, as suggested by Merrifield [8, p. 652]. In order to link this approach with the economic base model, however, we need to modify our model by making it respond to changes in exogenous demand. We accomplish this by expanding the interindustry portion of the model to include the household sector. Our expansion utilizes the vectors of labor income and consumption provided in Bourque and Conway [2, p. 50]. This has two effects. First, total employment,  $E_t$ , (generated as above), is now inclusive of intrahousehold or domestic employment. Second, by including the household column, we internalize local consumption and render the stimulus vector to be total final demand,  $Y_j$ , less the amount of local consumption; that is, the stimulus vector is now exogenous demand,  $Y_{ej}$ .

Traditionally this exogenous demand includes all government spending; that is, all government spending is from external sources. In this study, we are examining a state economy and therefore consider local and state government expenditures as internal (paid from inside sources). Thus, we have included in local consumption state and local government expenditures. The  $n+1$  diagonal term consists of the usual intrahousehold consumption and purchases from households by local and state governments. This is identical to the  $n+1$  entry on the labor income row vector. Merrifield correctly states "comparison of EB and I-O multipliers requires closure with respect to the household sector" [8, p. 652]. However, it is important that the definition of the household sector includes closure with respect to all local demand, including internal governments. Using this approach avoids Merrifield's extreme interpretation that a firm that trades directly or indirectly with external demand is totally basic [8]. The approach presented in this paper leaves federal government expenditures as exogenous.

Thus, the vector of total effects,  $G_j$ , is recast as:

$$G_j^* = Y_{ej}\pi_jK_j^* \text{ for } j = 1 \text{ to } (n+1) \tag{12}$$

where,  $K_j^*$  is the type II employment multiplier and  $Y_{ej}$  is the exogenous demand for products from sector  $j$ . The type II multiplier for sector  $j$  captures the direct, indirect, and induced effects of delivering a unit of  $j$ 's output to exogenous demand; it is somewhat unusual here because it includes local and state government expenditures. Generally, this is what the economic base multiplier tries to capture.

These notions may be integrated with the multiplier developed above, now over  $n+1$  sectors, as the multiplier  $M_2$ :

$$M_2 = \sum_j (Y_{ej} \pi_j K_j^*) / \sum_j (Y_{ej} \pi_j) \quad (13)$$

where the denominator is basic direct employment; that is, employment directly responding to external demand. The numerator is equal to total regional employment, directly and indirectly tied to the activities of the  $n+1$  sectors of the economy. In other words, the numerator may be measured from conventional employment data sources; the sector specific multipliers therefore are not needed to produce the base multiplier,  $M_2$ .

This multiplier also can be derived from the economic base model. In general form, total and basic employment are linearly related such that

$$E_T = a + bE_B, \quad (14)$$

where  $E_T$  is total employment,  $E_B$  is basic employment, and  $a$  and  $b$  are constants. This formulation is similar to Mulligan's except, in more traditional fashion, equation (14) relates total to basic employment rather than nonbasic to total [9, p. 5]. By differentiation of equation (14),  $b$  is equal to the ratio  $\Delta E_T / \Delta E_B$ . Thus, operating at the margin,  $b$  represents a base multiplier equivalent to  $M_2$ . This will be the appropriate multiplier to assess the impacts from changes in the basic sector.

Traditionally, base multipliers are estimated by dividing total employment by basic employment. This base multiplier can be equivalent to  $M_2$  only when the constant  $a$  is equal to zero; otherwise, it overestimates total effects when used for impact analysis. Mulligan refers to  $a$  as the autonomous component of employment [9, p. 5]. Specifically,  $a$  is that employment that is neither basic nor nonbasic; it is unrelated to the operation of the economic base. In this study the only autonomous employment reported is federal government employment; all other employment is tied to the local economy, directly and indirectly, through sales by sectors to exogenous final demand.

Therefore, to fully replicate the traditional employment base model, we must also include federal employment in total employment; changing equation (13) to:

$$M_2' = [\sum_j (Y_{ej}\pi_j K_j^*) + E_f] / [\sum_j (Y_{ej}\pi_j)] \quad (15)$$

This multiplier will be larger than  $M_2$  because only the numerator is increased. Because it brings income from the outside, however, federal government employment is commonly considered basic and is included with the basic direct:

$$M_2'' = [\sum_j (Y_{ej}\pi_j K_j^*) + E_f] / [\sum_j (Y_{ej}\pi_j) + E_f] \quad (16)$$

This final adjustment, in terms of employment, is similar to the adjustment presented by Billings in order to account for total sales by the labor sector to exogenous demand [1, p. 472].  $M_2''$  will be smaller than  $M_2'$ . Because  $M_2$  is always greater than one, it can be shown that  $M_2''$  will be smaller than  $M_2$  also.

Thus, we have defined and derived using an input-output structure a set of employment base multipliers that may be compared with estimates from other techniques such as those based upon location quotients. It is important to notice that such comparisons must consider the assignment of federal government employment.

### **Washington Employment Base Multiplier From Input-Output Data**

Using the 27 sector 1972 Washington state economic input-output tables [2], we calculated  $M_2$ ,  $M_2'$  and  $M_2''$  as in equations (13), (15), and (16):

$$M_2 = 3.13, M_2' = 3.32, \text{ and } M_2'' = 2.79.$$

The reader should notice from (17), which is simply an expansion of (13), the aggregate multiplier  $M_2$  is a weighted sum of the input-output multipliers.

$$\begin{aligned} M_2 = & [(Y_{e1}\pi_1)/\sum_j (Y_{ej}\pi_j)]K_1^* + \\ & [(Y_{e2}\pi_2)/\sum_j (Y_{ej}\pi_j)]K_2^* + \dots \\ & \dots + [(Y_{en+1}\pi_{n+1})/\sum_j (Y_{ej}\pi_j)]K_{n+1}^* \end{aligned} \quad (17)$$

The weight for each sector,  $j$ , is simply the ratio of its basic direct employment ( $Y_{ej}\pi_j$ ) created by exogenous demand ( $Y_{ej}$ ), divided by total basic direct employment ( $\sum_j Y_{ej}\pi_j$  over all  $n+1$  sectors). Table I presents

actual employment, basic direct employment, type II multipliers, and total employment effects for each sector.

Table I also presents the location quotient estimation of three base employment multipliers corresponding to those derived from the input-output model above. National figures were derived from data used in the 1972 national input-output study [4]. As usual, only those sectors with  $LQ > 1$  contribute to basic employment. Only 12 of 27 sectors make such a contribution, and these are generally underestimated by about 25 percent of the basic direct employment generated from the input-output table. Errors of this magnitude are often acceptable in the preparation of nonsurvey economic input-output models; however, the LQ approach also errs in a qualitative sense by its failure to identify sectoral contributions to basic employment in sectors where  $LQ < 1$ , contributing another underestimation error of about 27 percent. This compounds error in the estimation of the denominator of the economic base multiplier, producing values much too large (6.68, 7.08, and 5.06 from Table I). Isserman [7] and others have suggested that LQ estimates of base employment may be improved by working with finer levels of disaggregation. Accordingly, we calculated base employment using LQ ratios on 51 sectors, the finest disaggregation possible with the Washington data. The  $M_2$  type multiplier thus produced equaled 4.95, with similar corresponding sector allocations. The disaggregation schema for Washington carefully preserved the uniqueness of critical sectors.

## Conclusions

Aggregate employment base multipliers may be defined and determined through the economic input-output structure. The model above provides an outline of the derivation. Merrifield was correct when he noted "Use of appropriate definitions is the key... ." [8, p. 653]. This derivation and its definitions provide a benchmark measurement of the aggregate employment base multiplier for those investigating nonsurvey base multipliers.

The LQ approach assignment of employment to the basic sector works within acceptable levels for those sectors identified as export oriented. For those not so identified, more research is needed to establish the linkage of those sectors with exogenous activity. The modifications of Isserman and others should be noted in their attempt to reduce the severity of the bias of the location quotient [7, p. 39]. Among those suggestions are working at higher levels of disaggregation and possibly considering lower LQ thresholds. Now, we offer the idea of using input-output row marginals (a full table is not needed) and

employment data as an alternative for measuring basic economic activity. Recall here, for Tiebout the breakdown between direct and indirect was only possible by knowing the interindustry relations, a far more expensive undertaking [11]. Assuming  $Y_{ej}$ ,  $\pi_j$ , and  $E_j$  are known for each sector in an economy, the aggregate division into basic and nonbasic will be valid.

It should be noted that the sector specific difference between employment ( $E_j$ ) and basic direct employment ( $Y_{ej}\pi_j$ ) is the employment in sector  $j$  related to its local final demand plus intermediate demand from other local producers. In the sense that this is the difference between the total and basic employment, it is nonbasic in the tradition of the employment base model. However, it underscores a common problem of interpretation with the LQ approach; the nonbasic employment in a given sector is not the response to that sector's basic stimulus, but it is the response to total internal demand--including households, other industries, and government. The aggregate of all such differences is equal to total regional nonbasic employment, and it is only in the aggregate that it represents the response of the economy to the region's exogenous demand. If we wish to examine the sector specific response/stimulus relationship, we must examine a sector's total effect ( $G_j^*$ ) over its basic direct employment; that is, through the sectoral multiplier. But, if we know the employment, employment/output ratios, and external final demand, we can easily estimate base multipliers.

### **Endnote**

\*Both authors are with the Michigan State University department of geography.



## References

1. Billings, R. B., "The Mathematical Identity of the Multipliers Derived from the Economic Base Model and the Input-Output Model," *Journal of Regional Science*, 9 (1969), pp. 471-473.
2. Bourque, P. and R. Conway, *The Input-Output Structure of Washington State*, Graduate School of Business Administration, University of Washington, Seattle, WA (1976).
3. Bradley, I. and J. Gander, "Input-Output Multipliers: Some Theoretical Comments," *Journal of Regional Science*, 9, no. 2 (1969), pp. 309-317.
4. Crane, Jane-Ring F., "Employment and Employee Compensation in the 1972 Input-Output Study" (Washington: Bureau of Economic Analysis, Department of Commerce, 1981).
5. Hinojosa, R.C. and B.W. Pigozzi, "Estimation of Input-Output Coefficients by Means of Employment Data," *Environment and Planning A*, 14 (1982), pp. 1469-1478.
6. Hinojosa, R.C. and B.W. Pigozzi, "Updating Input-Output Tables using employment Data: A Reformulation," *Environment and Planning A*, 18 (1986), pp. 269-276.
7. Isserman, A.M., "The Location Quotient Approach to Estimating Regional Economic Impacts," *Journal of the American Institute of Planners*, 43 (1977), pp. 33-41.
8. Merrifield, J., "A Note on the General Mathematical Equivalency of Economic Base and Aggregate Input-Output Multipliers: Fact or Fiction," *Journal of Regional Science*, 27 (1987), pp. 651-654.
9. Mulligan, G., "Employment Multipliers and Functional Types of Communities: Effects of Public Transfer Payments," *Growth and Change*, 18, no. 3 (1987), pp. 1-11.
10. Pigozzi, B.W. and R.C. Hinojosa, "Regional Input-Output Inverse Coefficients Adjusted from National Tables," *Growth and Change*, 16 (1985), pp. 8-12.
11. Tiebout, C., *The Community Economic Base* (New York Committee for Economic Development. 1962).

**Table 1**  
Distribution of Employment, Washington State, 1972

Sector	Regional Employment (000's)	Distribution of Employment From I-O Model		Total Effect $G_j^*$	Allocation of Employment From LQ Approach	
		Basic Direct $Y_{ej\pi_j}$	Multiplier $K_j^*$		Location Quotient	Basic Employment
1	18.70	12.47	2.66	33.18	4.43	14.55
2	8.70	0.80	4.57	3.65	0.70	
3	32.60	16.88	2.36	39.85	9.75	29.19
4	8.30	0.60	2.43	1.47	0.73	
5	5.60	0.93	8.54	7.96	0.61	
6	14.40	10.42	4.93	51.36	1.64	5.64
7	7.00	3.62	5.04	18.28	0.59	
8	7.10	5.53	1.85	10.22	0.17	
9	47.80	31.06	3.57	110.79	4.51	37.19
10	8.60	6.81	3.95	26.91	7.25	7.41
11	17.80	13.72	4.45	61.04	1.57	6.48
12	11.60	1.71	2.52	4.31	0.63	
13	8.70	4.61	3.89	17.91	0.44	
14	5.60	0.44	3.59	1.57	0.51	
15	3.20	1.13	3.14	3.54	0.23	
16	10.10	8.25	4.03	33.29	1.65	3.98
17	7.70	3.35	3.14	10.53	0.30	
18	14.60	11.82	2.91	34.39	0.23	

Table 1 (continued)

Sector	Regional Employment (000's)	Distribution of Employment From I-O Model		Allocation of Employment From LQ Approach	
		Basic Direct $Y_{ej\pi j}$	Multiplier $K_j^*$	Location Quotient	Basic Employment
19	41.40	40.96	3.16	3.91	30.82
20	21.10	19.72	2.96	1.09	1.75
21	11.00	7.36	2.49	0.28	
22	72.20	48.59	3.44	0.84	
23	63.50	33.94	2.87	1.11	6.20
24	27.80	1.54	3.44	0.89	
25	296.70	83.89	2.11	1.14	37.39
26	69.30	17.57	2.78	0.99	
27	229.70	10.70	2.16	1.06	12.91
Subtotal	1070.80	398.41			193.51
Household	32.00	14.80	6.61		
Local Gov.	190.60				
Subtotal	1293.40	413.21			193.51
Fed. Gov.	77.60	77.60			77.60
Total	1371.00	490.81			271.11

From I-O:

 $M_2 = 1293.40/413.21$  $M_2' = 1371.00/413.21$  $M_2'' = 1371.00/490.81$ 

From LQ:

 $M_2 = 1293.40/193.51$  $M_2' = 1371.00/193.51$  $M_2'' = 1371.00/271.11$  $= 6.68$  $= 7.08$  $= 5.06$